Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 86622

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS –II

(Common to All Branches)

(Regulations 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $\sin 3t \cos 2t$.

2. Find the inverse Laplace transform of $\frac{e^{-\pi s}}{s^2+1}$.

3. Find a, b, c such that $\vec{F} = (3x + y + az)\,\vec{i} + (bx + 2y - z)\,\vec{j} + (3x + cy + 3z)\,\vec{k}$ is irrotational.

4. State Green's theorem in a plane.

5. If $u + iv$ is analytic, show that $v + iu$ is not an analytic.

6. Find the fixed points of the mapping $w = \frac{1+z}{1-z}$.

7. Evaluate $\int_1^b \int_1^a \frac{dx\ dy}{xy}$.

8. Evaluate $\int_0^3 \int_0^2 \int_0^1 dx\ dy\ dz$.

9. Identify and classify the singularity of $f(z) = \dfrac{\sin z}{z}$.

10. Find the residue of $f(z) = \dfrac{z+1}{(z-1)(z-2)}$ at $z = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using Laplace transform, Evaluate $\int_0^\infty e^{-t}\left(\dfrac{\cos at - \cos bt}{t}\right) dt$. (8)

(ii) Apply Laplace transform to solve $\left(D^3 - 3D^2 + 3D - 1\right) y = t^3 e^t$ given that $y(0) = 0,\ y'(0) = 0,\ y''(0) = -2$. (8)

Or

(b) (i) Apply convolution theorem to evaluate $L^{-1} \dfrac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$. (8)

(ii) Use transform method to solve $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + x = e^t$ with $x(0) = 2$ and $\dfrac{dx}{dt} = -1$ at $t = 0$. (8)

12. (a) (i) Prove that $\vec{F} = \left(y^2 \cos x + z^3\right)\vec{i} + (2y \sin x - 4)\vec{j} + \left(3xz^2 + 2\right)\vec{k}$ is irrational and find its scalar potential. (6)

(ii) Verify Stoke's theorem for $\vec{F} = x^2\vec{i} + xy\vec{j}$ in the square region in the xy- plane bounded by the lines $x = 0,\ y = 0,\ x = a,\ y = a$. (10)

Or

(b) (i) Prove that $\nabla^2\left(r^n \vec{r}\right) = n(n+3)\, r^{n-2}$. (6)

(ii) Verify Gauss divergence theorem for $\vec{F} = \left(x^2 - yz\right)\vec{i} + \left(y^2 - zx\right)\vec{j} + \left(z^2 - xy\right)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. (10)

13. (a) (i) If $f(z) = u + iv$ is an analytic function of prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ $(\log|f(z)|) = 0$. (8)

(ii) Find the image of the square whose vertices are $z = 1 + i$, $3 + i$, $1 + 3i$ and $3 + 3i$ under the transformation $w = \frac{1}{z}$. (8)

Or

(b) (i) Determine the analytic function $f(z) = u + iv$, $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

(ii) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $W = i, 0, \infty$. (8)

14. (a) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\ dy\ dz}{\sqrt{1 - x^2 - y^2 - z^2}}$. (8)

(ii) Find by double integration the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. (8)

Or

(b) (i) Evaluate by changing to polars, the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 y + y^3)\, dx\ dy$. (8)

(ii) Evaluate $\iiint_R \frac{1}{x^2 + y^2 + z^2}\, dx\ dy\ dz$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$. (8)

15. (a) (i) Evaluate $\int_0^\infty \frac{dx}{(1 + x^2)^2}$ using contour integration. (8)

(ii) Obtain Taylor's series for $f(z) = \frac{2z^3 + 1}{z(z + 1)}$ about $z = i$. (8)

Or

(b) (i) Using contour integration prove that $\int_0^\infty \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx = \frac{\pi}{6}$. (8)

(ii) Obtain Laurent's series expansion for $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ in the region $z < |Z| < 3$. (8)

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